# Exercises RESOLUTION IN PROPOSITIONAL LOGIC

**Symbols to use:**

¬

****☐****

**Exercise 1**

Using general resolution prove that the following formulas are theorems.

|  |  |
| --- | --- |
| 1. ; |  |
|  |  |
|  |  |
|  |  |

**Theoretical aspect used:**

**A propositional formula U is a theorem (tautology) iff the empty clause can be derived from the conjunctive normal form of ¬U, using the resolution algorithm.**

**U is a theorem (tautology) iff CNF(¬U) **☐****

**We set out to obtain the CNF of** ¬ by applying the normalization algorithm. Once we had this CNF, we considered the clauses within the CNF, and applied general resolution. We obtained the empty clause, and therefore we could conclude that is a theorem.

**Exercise 2**

Consider the following *hypotheses*:

 Mary will go to London this summer if both her friends Kate and Susan go.

 If Kate passes the English exam in May then she will go to London.

 Kate was in hospital from April until July and she didn’t take the English exam.

This summer Susan will go to London on a business trip.

and the *conclusion*:  Mary will go to London this summer.

Try to simplify the initial set of clauses by applying the transformations based on Davis-Putman procedure.

Using level-saturation strategy and the deletion strategy in propositional resolution check whether the following deduction holds:

**Theoretical aspect used:**

**Let propositional formulas. V iff iff CNF( ¬ **☐.****

****Davis-Putman procedure:****

* ****Delete clauses that are tautologies****
* ****Delete clauses subsumed by other clauses of S****
* ****Delete every clause that contains a pure literal (pure literal in set S of clauses: literal that appears in a clause of S, but its negation doesn’t appear in any clause of S)****
  + - * 1. **We transformed the hypotheses from natural language into propositional formulas, and then obtained the clauses we will be working with:**

= ¬K ¬S M

= ¬KE K

= ¬KE

= S

= ¬M

We applied the Davis-Putman procedure:

* We do not have any tautologies, no clauses were removed based on this transformation.
* We observed that subsumes , and therefore removed .
* After removing , we observed that we had a pure literal, ¬KE, and removed .

We finished last seminar with these 3 clauses, and planned to use level-saturation strategy and the deletion strategy.

= ¬K ¬S M

= S

= ¬M

**Theoretical aspect:**

**Level-saturation strategy**

**-first level: the initial set of clauses (S\_0 = {C\_1, C\_4, C\_5})**

**-S\_k = {Res(C\_i, C\_j) | C\_i belongs to S\_(k-1), C\_j belongs to reunion(S\_0, S\_1, ..., S\_(k-1)}**

**S\_1 = {Res(C\_i, C\_j) | C\_i belongs to S\_0, C\_j belongs S\_0}**

**Res(C\_1,C\_4)=** ¬K or M=C\_6

Res(C\_1,C\_5)=notK or notS=C\_7

S\_1={C\_6,C\_7}

**S\_2 = {Res(C\_i, C\_j) | C\_i belongs to S\_1, C\_j belongs S\_0 united with S\_1}**

**Res(C\_6,C\_5)=notK=C\_8**

**Res(C\_7,C\_4)=notK**

**S\_2={C\_8}**

**S\_3= {Res(C\_i, C\_j) | C\_i belongs to S\_2, C\_j belongs S\_0 united with S\_1 united with S\_2}**

**S\_3=empty set**

**The empty clause was not obtained, therefore the deduction does not hold.**

**Deletion strategy**

**Exercise 3. Party**

*Hypotheses*:

. Mary will go to the party if Lucy will go and George will not go.

. If John will go to the party then Lucy will go too.

. If John is in town he will go to the party.

. George is sick and can’t go to the party.

**.** Yesterday John has returned in town from Paris.

*Conclusion*: : Will Mary go to the party?

Try to simplify the initial set of clauses by applying the transformations based on Davis-Putman procedure.

Using general resolution in propositional logic check whether the following deduction holds: 

**Exercise 4**

Build a linear refutation from the following set of clauses:

1. ;

**Exercise 5**

Prove the consistency of the following sets of clauses using linear resolution.

1. 
2. 

**Exercise 6**

Using lock resolution prove the inconsistency of the following sets of clauses.

Choose two different indexings for the literals. For one indexing combine lock resolution with level-saturation strategy.

1. ;

**Exercise 7**

Check the consistency of the following sets of clauses using lock resolution.

Choose two different indexings for the literals:

1. ;
2. ;